Progression in addition and subtraction …. a guide for parents

Thorpe C of E Primary School 2021

Based on White rose Maths Hub document 2021

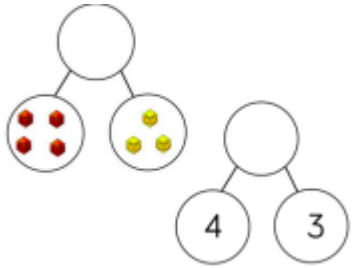
The following is the progression in calculations involving **addition** and **subtraction** which we follow in school.

It shows the **models** and **representations** we use with the children and which structures of addition and subtraction they support them to understand.

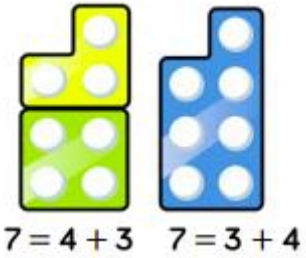
The **structures of addition** and the **structures of subtraction** are explained at the beginning of each section.

A further glossary of words can be found at the end.

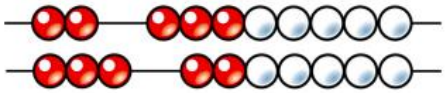
Manipulatives



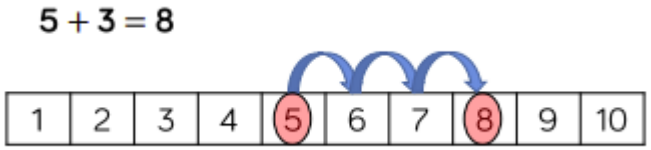
Part whole model



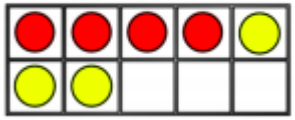
Numicon/Number shapes



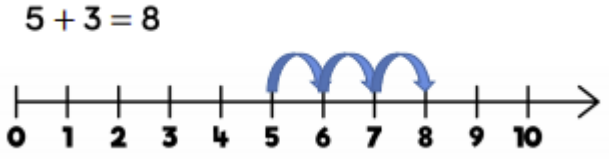
Bead String



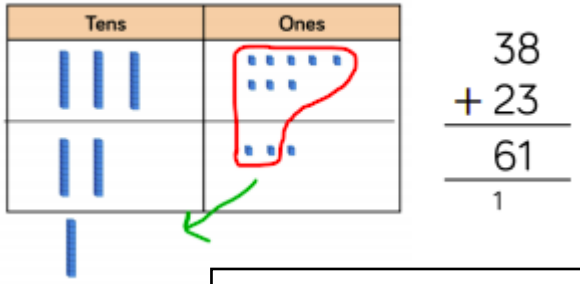
Number Track



Tens Frame

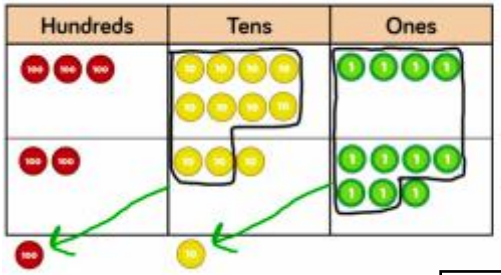


Number Line



$$\begin{array}{r} 38 \\ +\ 23 \\ \hline 61 \\ \scriptstyle 1 \end{array}$$

Diennes /Base ten



$$\begin{array}{r} 384 \\ +\ 237 \\ \hline 621 \\ \scriptstyle 1\ 1 \end{array}$$

Place Value counters

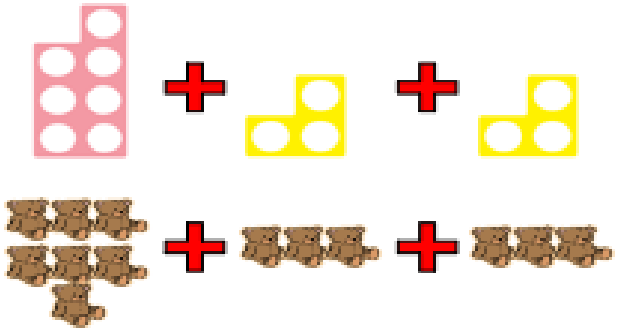
Addition

Structures of Addition (Haylock and Cockburn 2008)

Children should experience problems with all the different addition structures in a range of practical and relevant contexts e.g. money and measurement

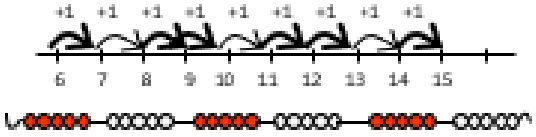
Aggregation

Union of two sets
How many/much altogether?
The total



Augmentation

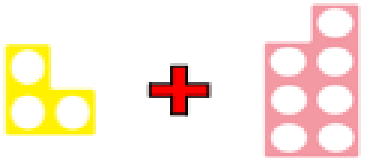
Start at and count on
Increase by
Go up by

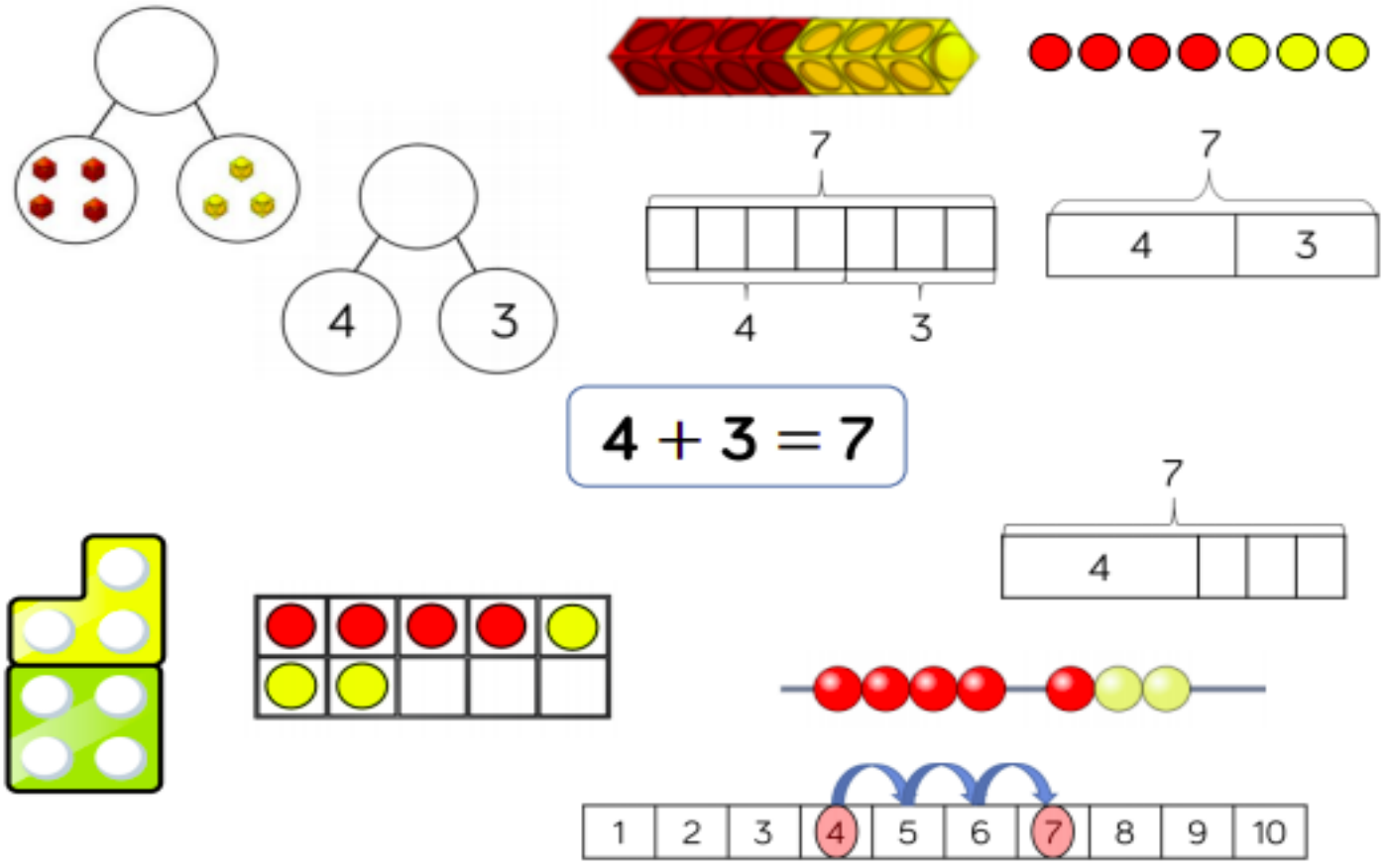


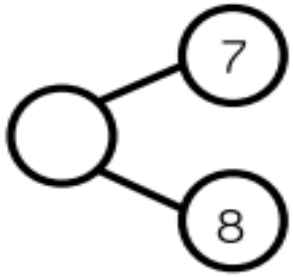
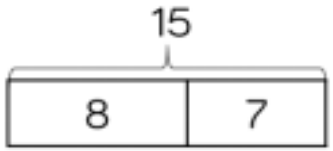
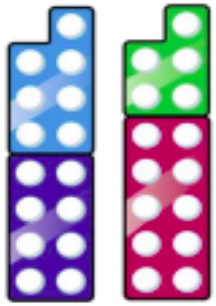
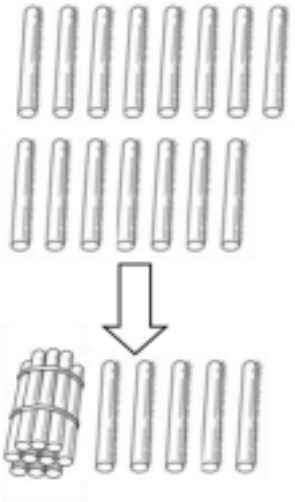
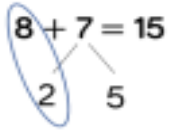
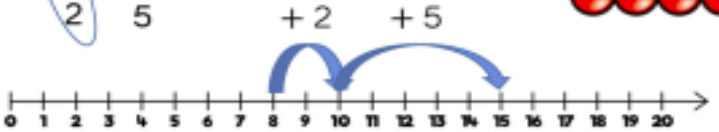
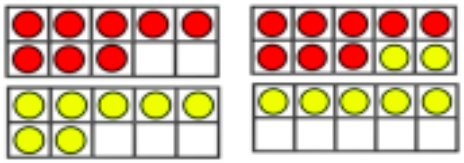
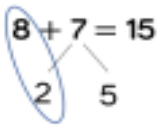
Commutative law

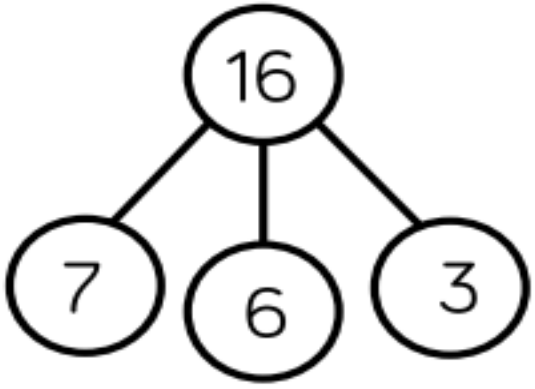
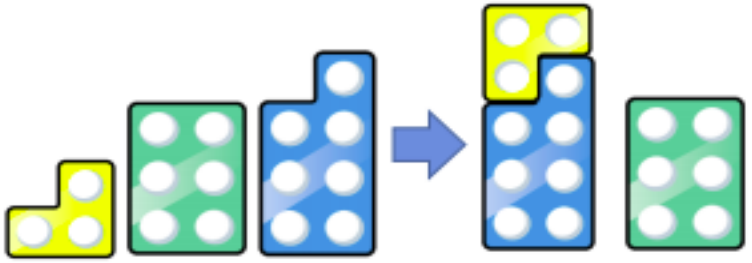
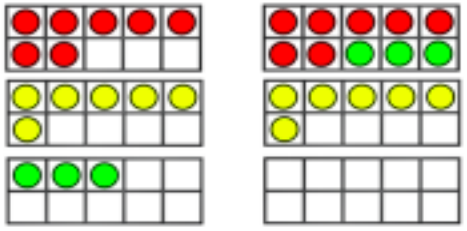
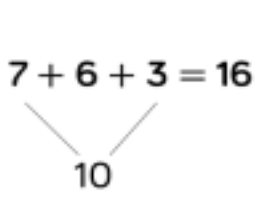
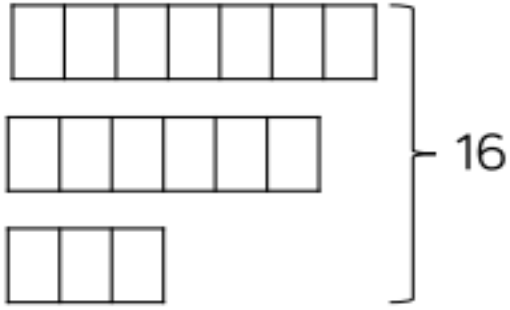
Understand addition can be done in any order
Start with bigger number when counting on
(Explain to children that subtraction does not have this property)

is the same as/equal to (=)



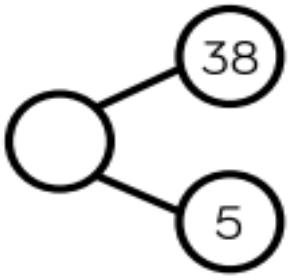
Skill: Add 1-digit numbers within 10	Year: 1
	When adding numbers to 10, children can explore both aggregation and augmentation. The part-whole model, discrete and continuous bar model, number shapes and ten frame support aggregation. The combination bar model, ten frame, bead string and number track all support augmentation.

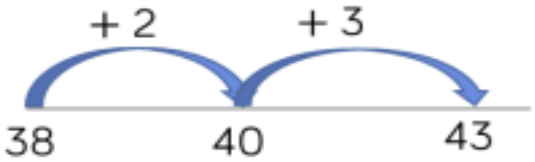
Skill: Add 1 and 2-digit numbers to 20	Year: 1/2
    $8 + 7 = 15$     	When adding one-digit numbers that cross 10, it is important to highlight the importance of ten ones equalling one ten. Different manipulatives can be used to represent this exchange. Use concrete resources alongside number lines to support children in understanding how to partition their jumps.

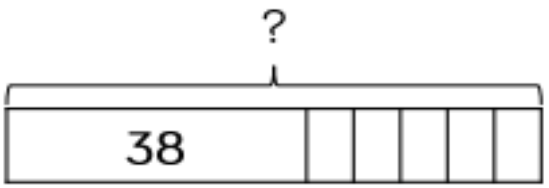
Skill: Add three 1-digit numbers	Year: 2
  $7 + 6 + 3 = 16$   	When adding three 1-digit numbers, children should be encouraged to look for number bonds to 10 or doubles to add the numbers more efficiently. This supports children in their understanding of commutativity. Manipulatives that highlight number bonds to 10 are effective when adding three 1-digit numbers.

Skill: Add 1-digit and 2-digit numbers to 100

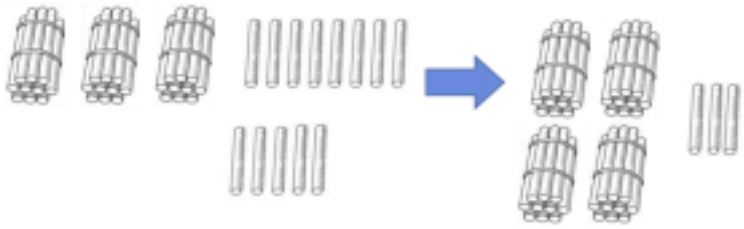
Year: 2/3







$$38 + 5 = 43$$



1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

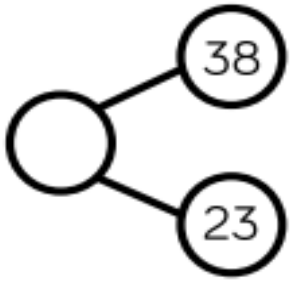
When adding single digits to a two-digit number, children should be encouraged to count on from the larger number.

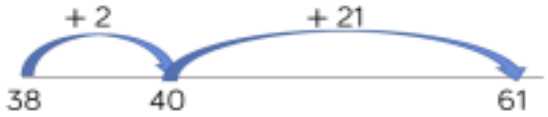
They should also apply their knowledge of number bonds to add more efficiently e.g. $8 + 5 = 13$ so $38 + 5 = 43$.

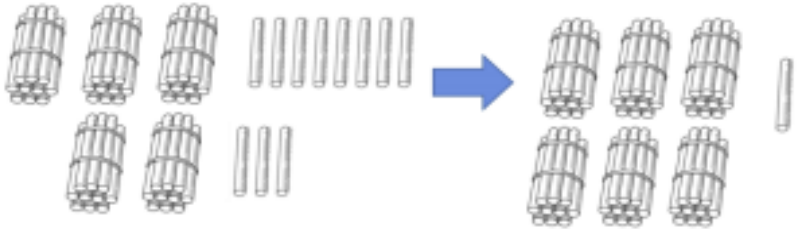
Hundred squares and straws can support children to find the number bond to 10.

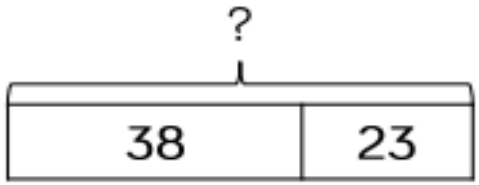
Skill: Add two 2-digit numbers to 100

Year: 2/3

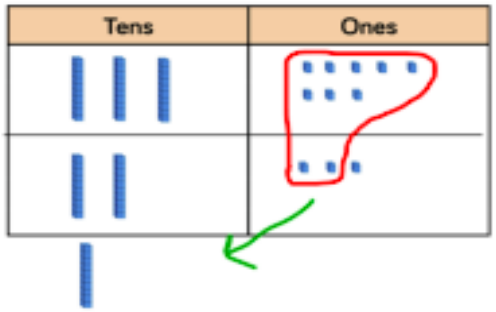








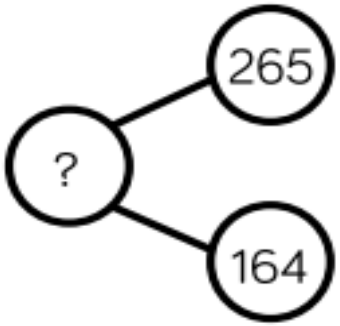
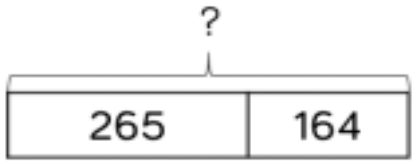
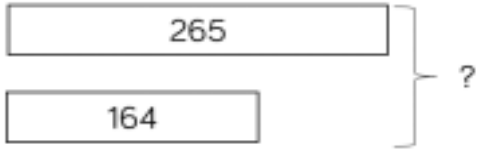
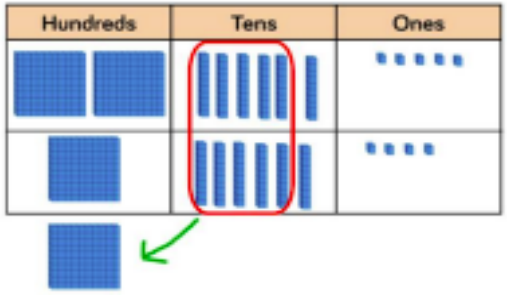
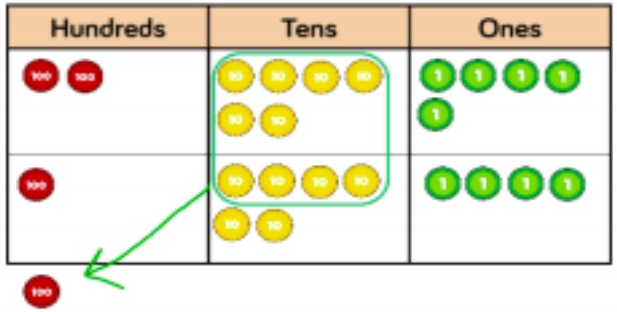
$$38 + 23 = 61$$



$$\begin{array}{r} 38 \\ +\ 23 \\ \hline 61 \\ \hline 1 \end{array}$$

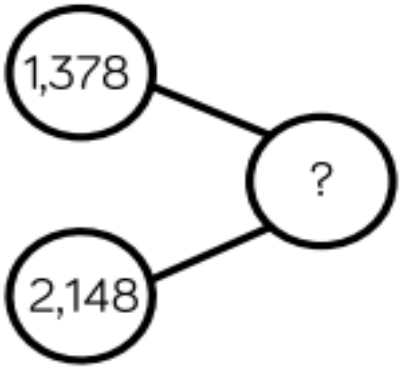
At this stage, encourage children to use the formal column method when calculating alongside straws, base 10 or place value counters. As numbers become larger, straws become less efficient.

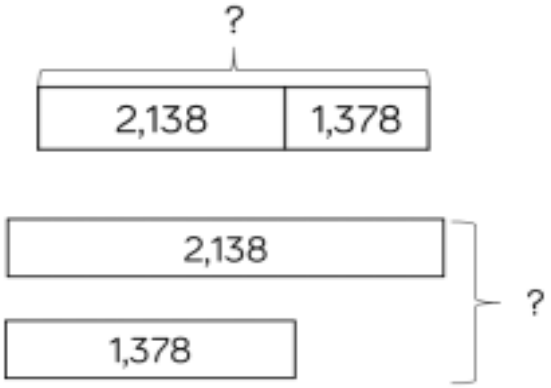
Children can also use a blank number line to count on to find the total. Encourage them to jump to multiples of 10 to become more efficient.

Skill: Add numbers with up to 3 digits	Year: 3
   $265 + 164 = 429$  265 + 164 429 1 	Base 10 and place value counters are the most effective manipulatives when adding numbers with up to 3 digits. Ensure children write out their calculation alongside any concrete resources so they can see the links to the written column method. Plain counters on a place value grid can also be used to support learning.

Skill: Add numbers with up to 4 digits

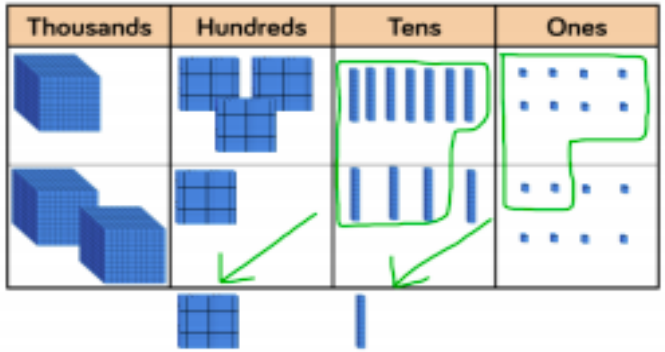
Year: 4

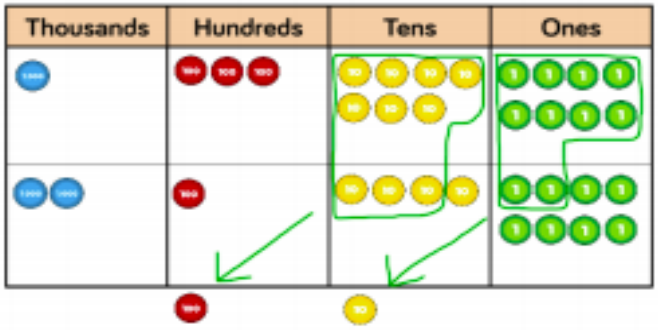




	1	3	7	8
+	2	1	4	8
	3	5	2	6
		1	1	

$$1{,}378 + 2{,}148 = 3{,}526$$





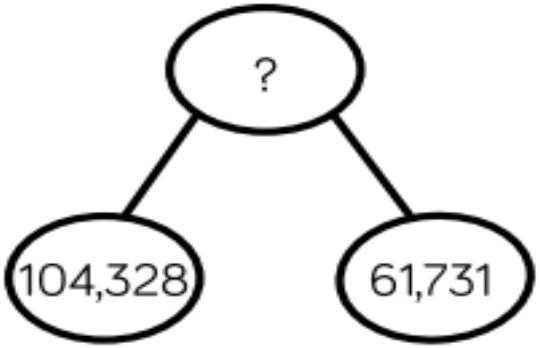
Base 10 and place value counters are the most effective manipulatives when adding numbers with up to 4 digits.

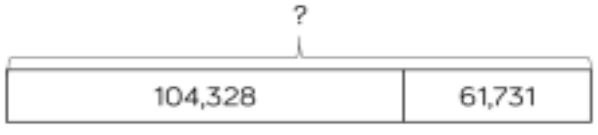
Ensure children write out their calculation alongside any concrete resources so they can see the links to the written column method.

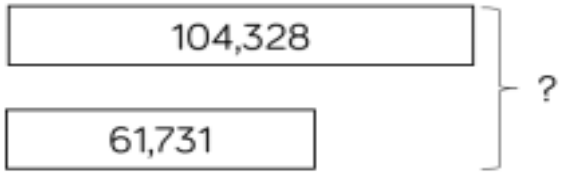
Plain counters on a place value grid can also be used to support learning.

Skill: Add numbers with more than 4 digits

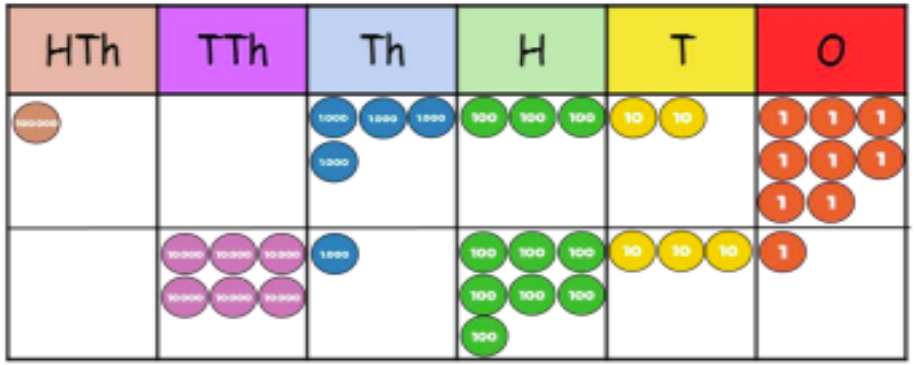
Year: 5/6







104,328 + 61,731 = 166,059



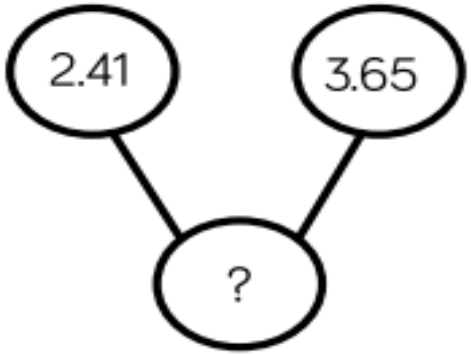
1	0	4	3	2	8
+	6	1	7	3	1
1	6	6	0	5	9
			1		

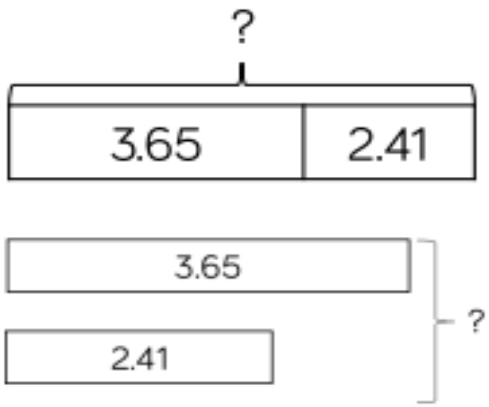
Place value counters or plain counters on a place value grid are the most effective concrete resources when adding numbers with more than 4 digits.

At this stage, children should be encouraged to work in the abstract, using the column method to add larger numbers efficiently.

Skill: Add with up to 3 decimal places

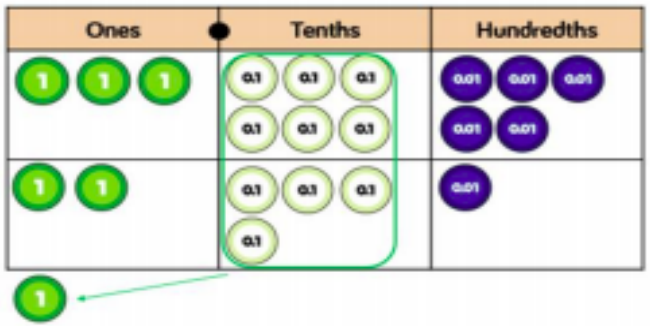
Year: 5

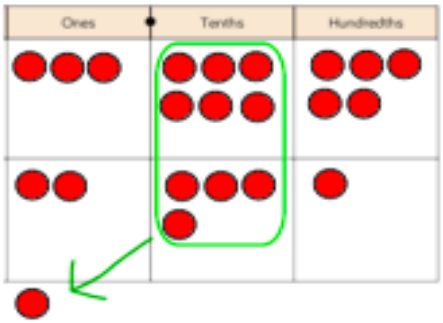




$$\begin{array}{r} 3.65 \\ +\ 2.41 \\ \hline 6.06 \\ \hline 1 \end{array}$$

$$3.65 + 2.41 = 6.06$$





Place value counters and plain counters on a place value grid are the most effective manipulatives when adding decimals with 1, 2 and then 3 decimal places.

Ensure children have experience of adding decimals with a variety of decimal places. This includes putting this into context when adding money and other measures.

Subtraction

Structures of Subtraction (Haylock and Cockburn 2008)

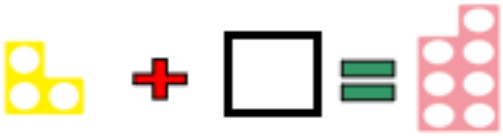
Children should experience problems with all the different subtraction structures in a range of practical and relevant contexts e.g. money and measurement

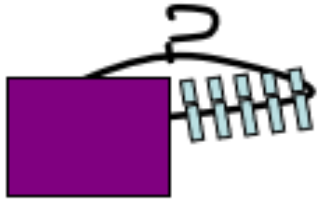
Partitioning

Take away
... how many left?
How many are not?
How many do not?

Inverse-of-addition

What must be added?
How many (much) more needed?





There are ten pegs on the hanger – how many are covered?

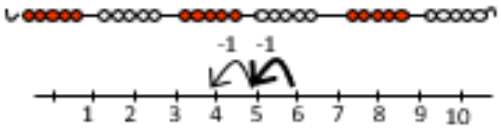
Comparison

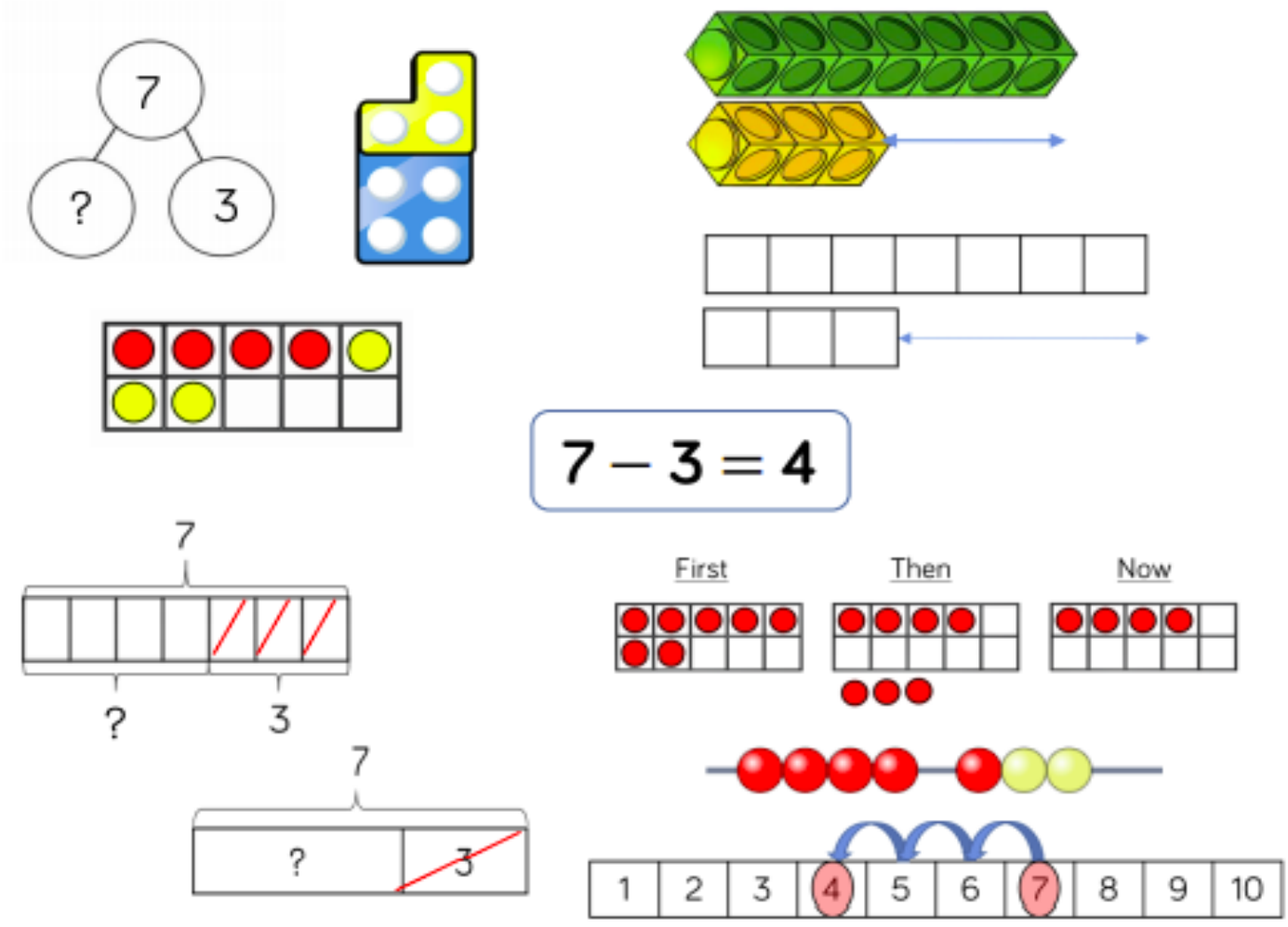
What is the difference?
How many more?
How many less (fewer)?
How much greater?
How much smaller?

'two more than three is five or two less than five is three'

Reduction

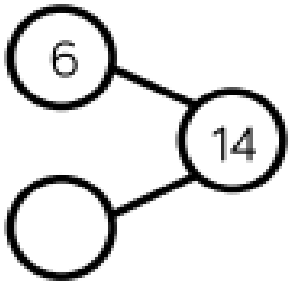
Start at and reduce by
Count back by
Go down by

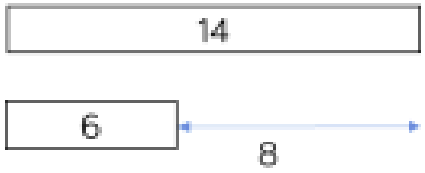


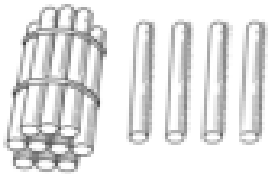
Skill: Subtract 1-digit numbers within 10	Year: 1
	Part-whole models, bar models, ten frames and number shapes support partitioning. Ten frames, number tracks, single bar models and bead strings support reduction. Cubes and bar models with two bars can support finding the difference.

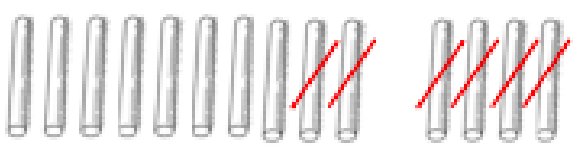
Skill: Subtract 1 and 2-digit numbers to 20

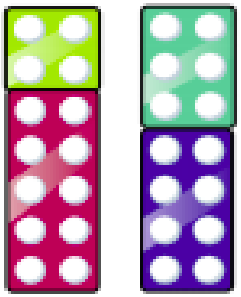
Year: 1/2



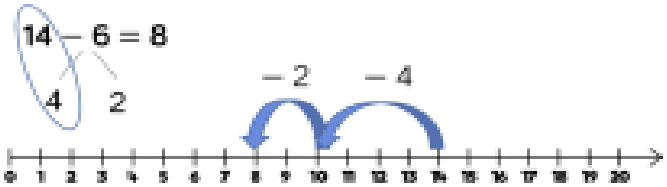


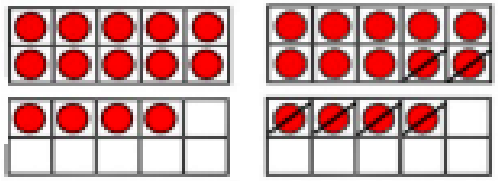


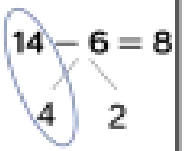




14 − 6 = 8





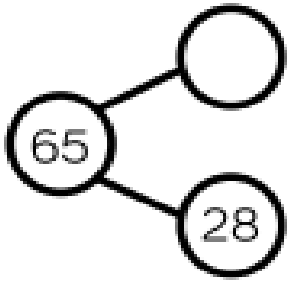


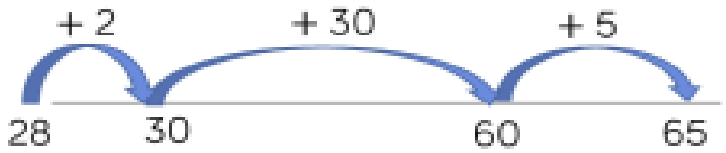
When subtracting one-digit numbers that cross 10, it is important to highlight the importance of ten ones equalling one ten.

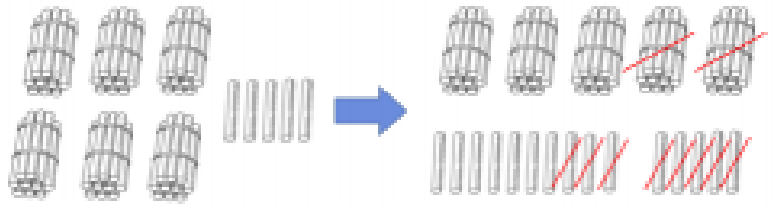
Children should be encouraged to find the number bond to 10 when partitioning the subtracted number. Ten frames, number shapes and number lines are particularly useful for this.

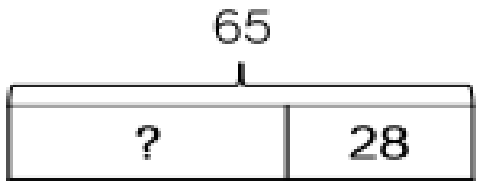
Skill: Subtract 1 and 2-digit numbers to 100

Year: 2

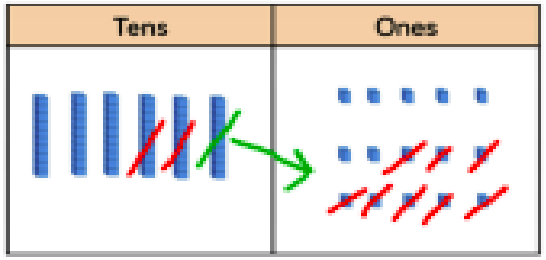




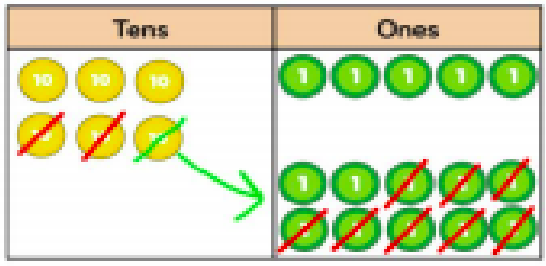




$$65 - 28 = 37$$



$$\begin{array}{r} {}^{5}\not{6}\,{}^{1}\not{5} \\ -\ 2\ 8 \\ \hline 3\ 7 \end{array}$$

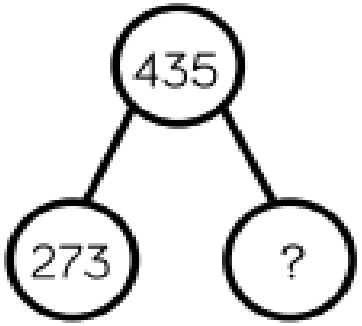


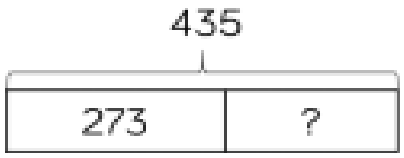
At this stage, encourage children to use the formal column method when calculating alongside straws, base 10 or place value counters. As numbers become larger, straws become less efficient.

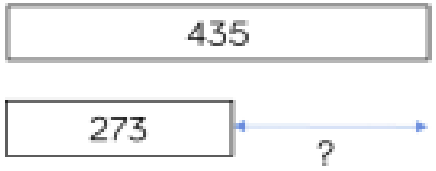
Children can also use a blank number line to count on to find the difference. Encourage them to jump to multiples of 10 to become more efficient.

Skill: Subtract numbers with up to 3 digits

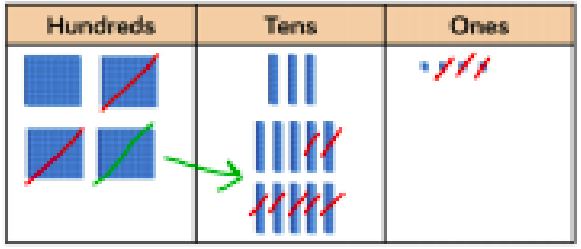
Year: 3



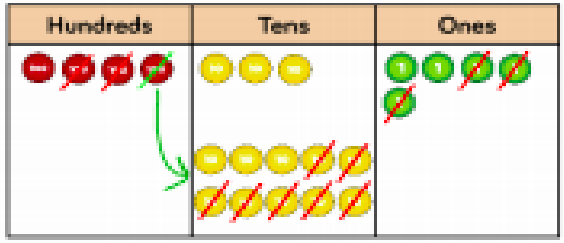




$$435 - 273 = 262$$



$$\begin{array}{r} {}^{3}\not{4}{}^{1}\not{3}5 \\ -\ 273 \\ \hline 262 \end{array}$$



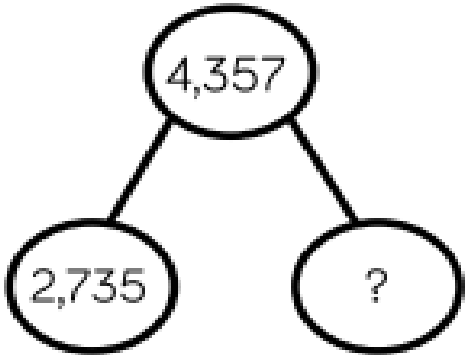
Base 10 and place value counters are the most effective manipulative when subtracting numbers with up to 3 digits.

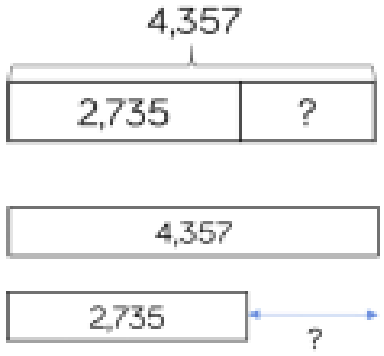
Ensure children write out their calculation alongside any concrete resources so they can see the links to the written column method.

Plain counters on a place value grid can also be used to support learning.

Skill: Subtract numbers with up to 4 digits

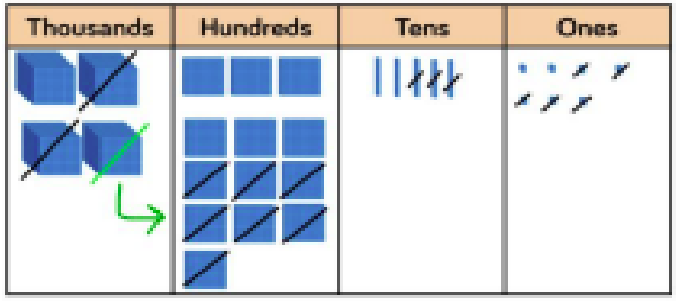
Year: 4

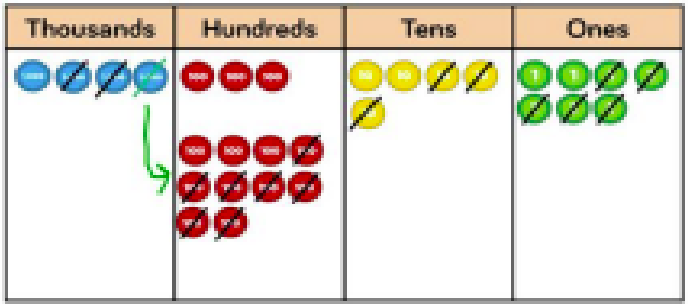




$$\begin{array}{r} {}^{3}\cancel{4}^{1}357 \\ -\ 2735 \\ \hline 1622 \end{array}$$

$$4{,}357 - 2{,}735 = 1{,}622$$





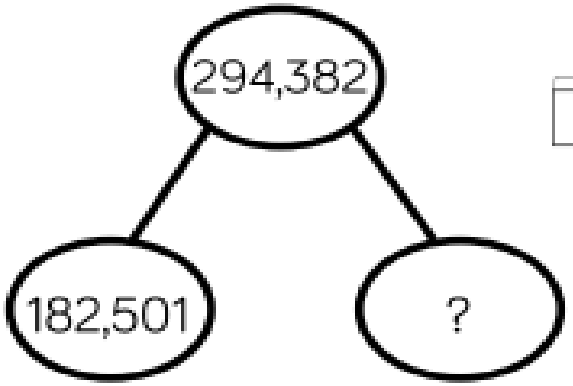
Base 10 and place value counters are the most effective manipulatives when subtracting numbers with up to 4 digits.

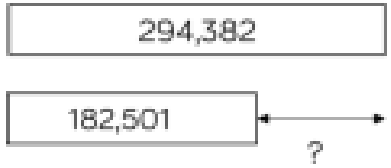
Ensure children write out their calculation alongside any concrete resources so they can see the links to the written column method.

Plain counters on a place value grid can also be used to support learning.

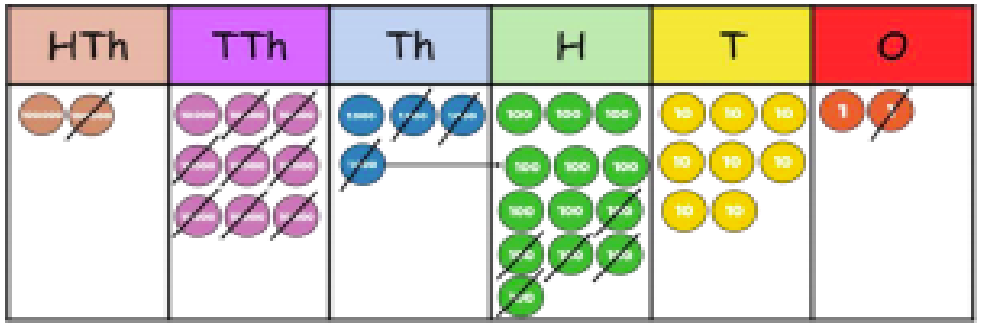
Skill: Subtract numbers with more than 4 digits

Year: 5/6





294,382 − 182,501 = 111,881



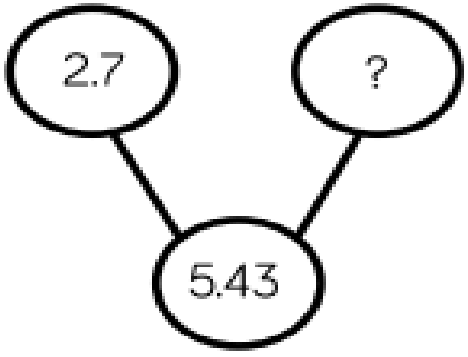
	2	9	$^{3}\not{4}$	$^{1}3$	8	2
−	1	8	2	5	0	1
	1	1	1	8	8	1

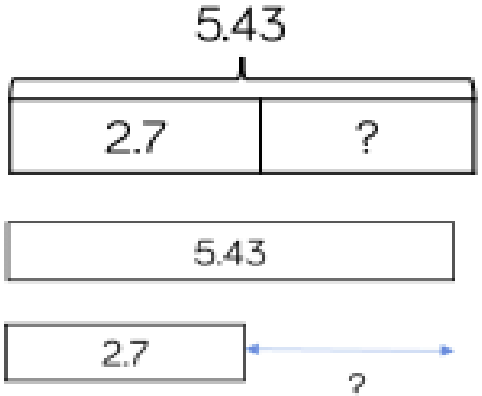
Place value counters or plain counters on a place value grid are the most effective concrete resource when subtracting numbers with more than 4 digits.

At this stage, children should be encouraged to work in the abstract, using column method to subtract larger numbers efficiently.

Skill: Subtract with up to 3 decimal places

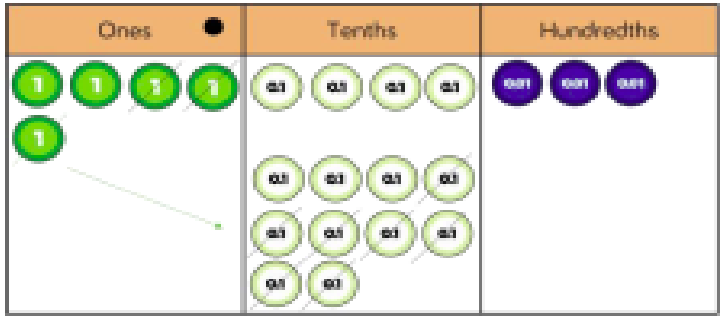
Year: 5

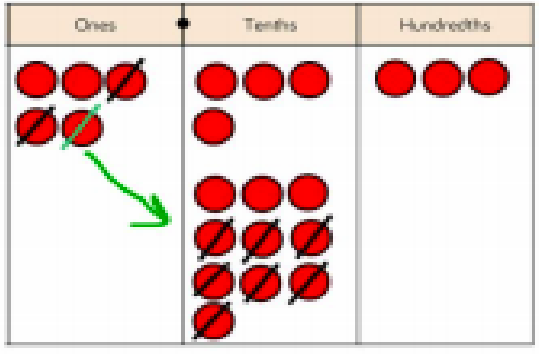




$$\begin{array}{r} {}^{4}\cancel{5}.{}^{1}43 \\ -\ 2.7 \\ \hline 2.73 \end{array}$$

$$5.43 - 2.7 = 2.73$$





Place value counters and plain counters on a place value grid are the most effective manipulative when subtracting decimals with 1, 2 and then 3 decimal places.

Ensure children have experience of subtracting decimals with a variety of decimal places. This includes putting this into context when subtracting money and other measures.

Glossary

Addend - A number to be added to another.

Aggregation - combining two or more quantities or measures to find a total.

Augmentation - increasing a quantity or measure by another quantity.

Commutative – numbers can be added in any order.

Complement – in addition, a number and its complement make a total e.g. 300 is the complement to 700 to make 1,000

Difference – the numerical difference between two numbers is found by comparing the quantity in each group.

Exchange – Change a number or expression for another of an equal value.

Minuend – A quantity or number from which another is subtracted.

Partitioning – Splitting a number into its component parts.

Reduction – Subtraction as take away.

Subitise – Instantly recognise the number of objects in a small group without needing to count.

Subtrahend - A number to be subtracted from another.

Sum - The result of an addition.

Total – The aggregate or the sum found by addition.